A COMPLEX ADAPTIVE NOTCH FILTER BASED ON THE STEIGLITZ-MCBRIDE METHOD

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ABSTRACT

This paper propose a new complex adaptive notch Þlter (ANF) structure based on the Steiglitz-McBride (SM) method. Recursive least square (RLS) algorithm is applied to the proposed ANF with optimized stepsize. Simulations show that RLS-SM ANF converges fast and requires less computational complexity than the conventional ANF using recursive prediction error (RPE) algorithm.

1. INTRODUCTION

Adaptive notch Þlters (ANF) are widely used in many signal processing applications to extract, eliminate or trace narrowband or sinusoidal signals embedded in broadband noise [2]. If such signal consists of in-phase and quadrature components, a complex coefÞcient ANF must be implemented. Most of such applications are in radar and communication systems. An early contribution to ANF algorithms by Nehorai [3] imposed constraints on a notch transfer function, which leads to simple relations between poles and zeros in adaptive Þlter design. Nehorai also derived the Gauss-Newton type recursive prediction error (RPE) algorithm [3] whose structure is shown in Fig. 1. The algorithm adjusts the Þlter coefÞcients to minimize the cost function $E\{|e(k)^2|\}$ by calculating the gradient recursively. Based on the same objective function, Pei [4] extended the RPE algorithm to complex coefÞcient ANF, which converges to a small biased solution. Cheng [1] derived a new real-valued ANF algorithm using the well-known SM method. ChengÕs idea comes from the system identiÞcation application by using delayed signal as the reference signal [7]. In this paper, we extend the result of [1] and derive complex coefÞcient adaptive notch Þlter algorithm using SM method. Furthermore, optimized stepsize is employed in our algorithm. Simulation results show that the complex SM method converge faster than RPE algorithms in [4].

The paper is organized as follows. In Section 2, Complex ANF algorithms using the SM method are derived. Section 3 analyzes the ANF convergence. In Section 4, simulation results show the improved performance of ANF using SM method. Finally, Section 6 concludes the paper.

2. SYSTEM MODEL

Consider a measured stationary data $y(k)$ which comprises a known number of complex sinusoids and a white noise $\epsilon(k)$,

$$y(k) = \sum_{i=1}^{M} R_i exp(j\omega_i k + \phi) + \epsilon(k) \tag{1}$$

where the amplitudes $\{R_i\}$, phases $\{\phi_i\}$ and the frequencies $\{\omega_(k)\}$ are unknown constants. $\epsilon(k)$ is a sequence of i.i.d. complex random variable with zero mean and variance denoted by σ^2. It is known that (1) can be represented by an ARMA model [6],

$$A(q^{-1})y(k) = A(\rho q^{-1})\epsilon(k) \tag{2}$$

where $A(q^{-1})$ is a monic polynomial of order M and its roots are on the unit circle with arguments equal to $\{\omega_i\}$. The parameter $\rho \in (0,1)$ is a pole radius which keeps the Þlter $A(q^{-1})/A(\rho q^{-1})$ stable. Such Þlter is also known as constrained form notch Þlter.

3. COMPLEX SM ANF ALGORITHMS

The idea of ANF algorithm using the SM method comes from the system identiÞcation application by using delayed signal as the reference signal [7]. The resulting block diagram is depicted in Fig. 2. The function of the delay factor Δ in the Þgure is to decorrelate the the preÞlter outputs $g(k)$ and $h(k)$ in the upper and lower paths. If the noise is white,$\Delta = 1$ is enough to decorrelate the signals. By letting the structure to approximate a notch Þlter, the structure shown in Fig. 3 is obtained.

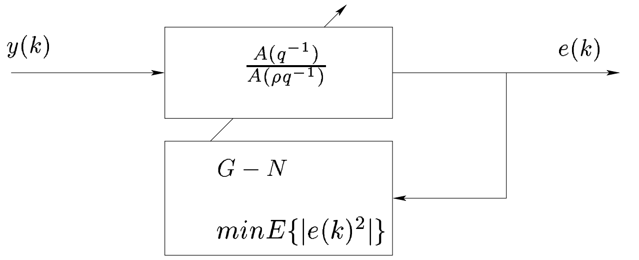

Figure 1: Adaptive notch Þlter with recursive prediction error algorithm for coefÞcient adjustment

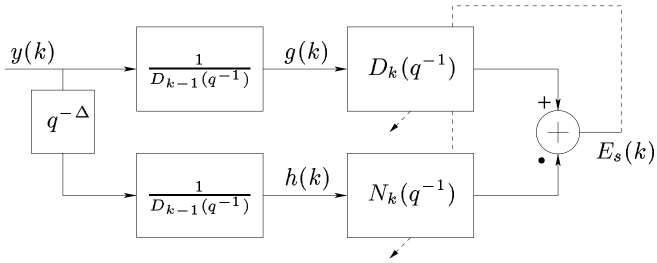

Figure 2: Adaptive notch Þlter based on SM method

3.1. Simplified ANF structure

Assuming the broadband signal is stationary, we can move the delay operation at the lower branch after the preÞlter shown in Fig. 2, such arrangement can save one preÞlter block. Larger Δ can be chosen in other applications when the noise is colored. Since the resulting transfer function after the convergence is desired to be a notch Þlter, the following equation should be satisÞed:

$$\lim_{k\to\infty}\left(1-q^{-1}\frac{N_k(q^{-1})}{D_k(q^{-1})}\right)=\frac{A(q^{-1})}{A(\rho q^{-1})} \quad (3)$$

This yields the block diagram shown in Fig 4. Therefore the polynomials $D_k(\rho q^{-1})$ and $N_k(q^{-1})$ in modiÞed Fig. 2 can be deÞned as

$$\begin{aligned} D_k(q^{-1}) &= A(\rho q^{-1}) \\ N_k(q^{-1}) &= [A(\rho q^{-1})-A(q^{-1})]q \end{aligned} \quad (4)$$

3.2. Algorithm derivation

The adaptive algorithm can be derived directly from Fig 4. Let the estimated coefÞcient vector $\mathbf{\Theta}_{k-1}=[a_1,a_2,\cdots,a_M]^T_{k-1}$ where the superscript T denotes the transpose operation. Using the recursive least square (RLS) procedure, we derive the detailed algorithm as

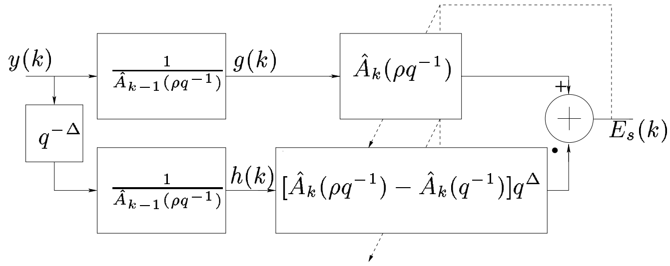

Figure 3: Block diagram of ANF using SM method in adaptive line enhancer structure

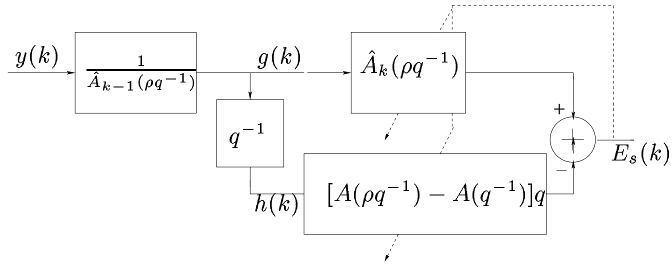

Figure 4: Block diagram of modiÞed ANF using SM method

below.

Step1: PreÞlter

$$g(k)=\frac{1}{\hat{A}_{k-1}(\rho q^{-1})}y(k) \quad (5)$$

where $\hat{A}_{k-1}(\rho q^{-1})=1+a^*_{1,k-1}\rho q^{-1}+a^*_{2,k-1}\rho^2q^{-2}+\cdots+a^*_{M,k-1}\rho^Mq^{-M}$. Rearranging the input-output, we obtain,

$$g(k)=y(k)-\mathbf{\Theta}^H_{k-1}\mathbf{G}_k \quad (6)$$

where the superscript H denotes conjugate transpose, and $\mathbf{G}_k=[\rho g(k-1),\rho^2g(k-2),\cdots,\rho^Mg(k-M)]^T$. Since the preÞlter output for lower branch $h(k)=g(k-1)$ is a delayed version of $g(k)$, one preÞlter can be saved.

Step 2: Output expression
The output can also be arranged in vector form,

$$\begin{aligned} \epsilon(k) &= g(k)\hat{A}_k(q^{-1})-h(k+1)[\hat{A}_k(\rho q^{-1})-\hat{A}_k(q^{-1})] \\ &= g(k)-\mathbf{\Theta}^H_{k-1}\mathbf{\Phi}(k) \end{aligned} \quad (7)$$

where $\mathbf{\Phi}(k)=[\phi_1(k),\phi_2(k),\cdots,\phi_M(k)]^T$ and $\phi_i(k)=-\rho^ig(k-i)+(\rho^i-1)h(k-i+1)$.
Step 3: Covariance matrix update

$$\mathbf{P}(k+1)=\frac{1}{\lambda(k)}\left[\mathbf{P}(k)-\frac{\mathbf{P}(k)\mathbf{\Phi}(k)\mathbf{\Phi}^H(k)\mathbf{P}(k)}{\frac{\lambda(k)}{\alpha(k)}+\mathbf{\Phi}^H(k)\mathbf{P}(k)\mathbf{\Phi}(k)}\right] \quad (8)$$

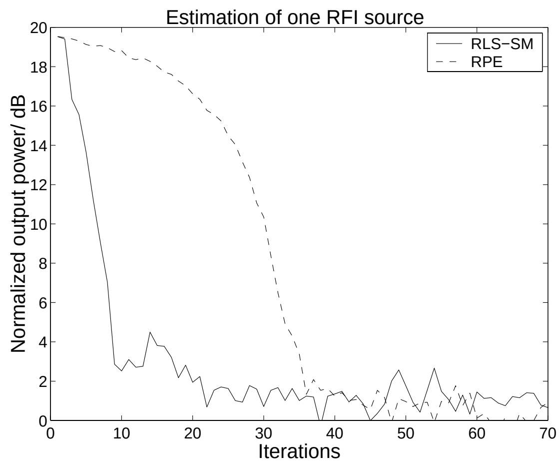

Figure 5: The output MSE when estimating 1 sinusoid

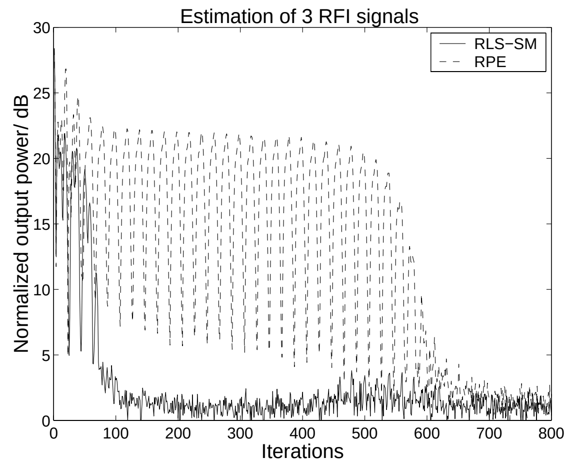

Figure 6: the output MSE when estimating 3 sinusoids(Þxed α and ρ for RPE)

where $\lambda(k) = 1 - \alpha(k)$ is the forgetting factor in the RLS algorithm.

Step 4: Estimation parameter update

$$\mathbf{\Theta}(k+1) = \mathbf{\Theta}(k) + \alpha(k)\mathbf{P}_{k+1}\mathbf{\Phi}(k)\epsilon(k)^* \qquad (9)$$

The algorithm is in the RLS form. The differences between the RPE algorithm and our RLS algorithm using the SM methods are on the choice of regression vector $\mathbf{\Phi}(k)$ and error $\epsilon(k)$. A better choice of these parameters can lead to faster convergence and less excess mean square error (MSE) at the output.

4. CONVERGENCE CONSIDERATIONS

In both RPE and RLS-SM algorithms, the convergence speed and the excess MSE depends on two parameters: the pole radius ρ and the stepsize α.

4.1. Time-varying ρ

In most of the ANF algorithms the pole radius is a time-varying function [4, 1]. The reason is that ρ determines the bandwidth of the notches. Practically, if no *a priori* information is available on the input sinusoid, when the notches are too narrow, the algorithm may not converge. On the other hand, a larger pole radius ($\rho \to 1$) will lead to less excess MSE after convergence. Therefore an exponential function is often used for $\rho \to 1$ by letting ρ grow from an initial value $\rho(1)$ to the desired value $\rho(\infty)$ according to

$$\rho(k+1) = \rho_0\rho(k) + (1-\rho_0)\rho(\infty) \qquad (10)$$

where ρ_0 determines the rate of change in $\rho(k)$.

4.2. Optimal α for SM method

In the algorithms derived by Pei [4] and Cheng [1], the stepsize is treated in the same way as ρ, which approaches exponentially the predeÞned value. Without *a priori* knowledge of the input, the choice of α is usually a difÞcult task. In this paper, we apply the optimal stepsize derived in [5] for IIR Þlters using the SM method. An optimal stepsize puts a proper weight on the new incoming data at each updating step, which will lead to the maximal reduction of MSE, thus speeding up the convergence. The optimal α has the form

$$\alpha(k) = \frac{\kappa}{1+\tau(k)} \qquad (11)$$

where $0 < \kappa < 1$ is a reduction factor which is related to the Þlter order and $\tau(k) = \mathbf{\Phi}^H(k)\mathbf{P}(k)\mathbf{\Phi}(k)$. Note that $\tau(k)$ is an intermediate result of (8), so that Þnding the optimal convergence factor does not increase the complexity of the algorithm.

5. SIMULATIONS

We apply the proposed complex RLS-SM ANF and RPE algorithm to suppress sinusoid in white noise. In all the following experiments, The pole radius is time-varying according to (10), where $\rho_0 = 0.99$, $\rho(1) = 0.7$ and $\rho(\infty) = 0.995$. We use optimal stepsize derived in (11) for RLS-SM algorithms, whereas the optimal stepsize is used in RPE algorithm. The input signal is modelled as in (1). The sinusoid and white noise are chosen such that the noise is 20 dB below the sinusoidal level. The output signal is normalized with respect to the white noise power, in other words, 0 dB output is the

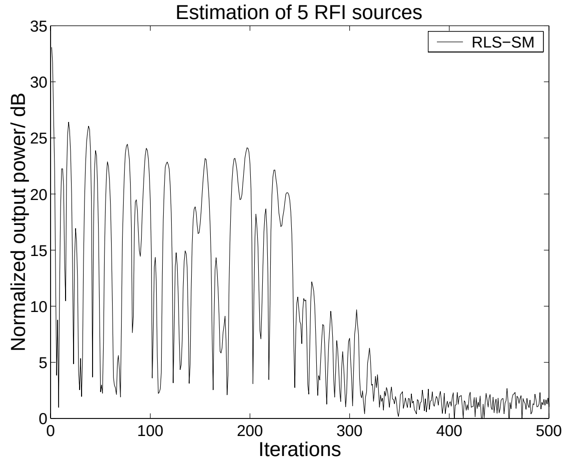

Figure 7: The output MSE when estimating 5 sinusoids using RLS-SM algorithm

best suppression result that we can achieve. The simulation results are averaged over 100 independent runs.

5.1. The first-order notch filter

There is one complex sinusoid signals embedded in white noise, whose frequency is $\omega_1 = 0.015$. The Þlter output MSE is shown in Fig. 5. It can be seen that using RLS-SM algorithm leads to the optimal solutions in ca.15 iterations, whereas the RPE algorithm requires ca.40 iterations to converge.

5.2. The third-order notch filter

This experiment is for the case of three sinusoids where their frequencies are $\omega_1 = 0.1$, $\omega_2 = 0.2$ and $\omega_3 = 0.4$. In this case, RPE algorithm with time-varying pole radius can not converge, therefore ρ is set Þxed at 0.8. As can be seen in Fig. 6, the RLS-SM algorithm works very well, whereas the RPE algorithm converges much more slowly and generates higher excess MSE.

5.3. Estimation of 5 sinusoids using RLS-SM algorithm

This is an extreme case that there are 5 sinusoids exist. where their frequencies are $\omega_1 = 0.1$, $\omega_2 = 0.2$, $\omega_3 = 0.25$, $\omega_4 = 0.3$ and $\omega_5 = 0.4$ respectively. Since this is a difÞcult situation for ANF to converge, we loose the criteria on extra MSE and let $\rho(\infty) = 0.95$. As can be seen in Fig. 7, the algorithm converges in ca. 400 iterations, whereas RPE algorithm fails to converge within 1024 iterations, even when the optimal stepsize is utilized.

6. CONCLUSION

In this paper, the ANF using SM method proposed by Cheng [1] is extended to the complex-coefÞcient case. We propose a simpliÞed structure by relocating the delay elements on one branch of the Þlter assuming the broadband process is white and stationary. Simulations show that the RLS-SM algorithm converges faster than RPE algorithm when suppressing sinusoid embedded in white Gaussian noises. Our algorithm is more robust compared with the RPE algorithm since it can deal with up to 5 sinusoids. Furthermore, optimized stepsize developed in [5] for RLS-SM algorithms is also employed to speed up the convergence.

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