EQUALIZATION AND MODELING OF AUDIO SYSTEMS USING KAUTZ FILTERS

Tuomas Paatero and Matti Karjalainen

Helsinki University of Technology
Laboratory of Acoustics and Audio Signal Processing
P. O. Box 3000, FIN-02015 HUT
FINLAND
tuomas.paatero@hut.fi

ABSTRACT

This paper demonstrates the applicability of Kautz filters in audio signal processing. New methods for the choosing of Kautz filter poles are presented and utilized in two audio oriented applications.

1. INTRODUCTION

Frequency warping using allpass structures or *Laguerre filters* [7] has found increasingly applications in audio signal processing due to good match with the auditory frequency resolution [3, 8]. *Kautz filters* [6, 2] can be seen as a further generalization where each transversal element may be different, including complex conjugate poles. This enables arbitrary allocation of frequency resolution for filter design, such as modeling and equalization (inverse modeling) of linear systems.

After a brief theoretical background of implementation and design principles, we present two examples as case studies of using Kautz filters in modeling and inverse modeling of audio systems. In the first case we apply the method to loudspeaker response equalization. The second case deals with the modeling of guitar body impulse response.

2. KAUTZ FUNCTIONS AND FILTERS

For a given set of desired poles $\{z_i\}$ in the unit disk, the corresponding set of *rational orthonormal functions* is uniquely defined in the sense that the lowest order rational functions, square-integrable and orthonormal on the unit circle, analytic for $|z| > 1$, are of the form [9]

$$G_i(z) = \frac{\sqrt{1 - z_i z_i^*}}{z^{-1} - z_i^*} \prod_{j=0}^{i} \frac{z^{-1} - z_j^*}{1 - z_j z^{-1}}, \quad i = 0, 1, \ldots. \qquad (1)$$

A Kautz filter is a finite weighted sum of functions (1), which reduces to a transversal structure of Fig. 1. Defined in this manner, Kautz filters are merely a class of fixed-pole IIR filters, forced to produce orthonormal tap-output impulse responses. However, the fact that functions (1) provide a (*Fourier*) basis representation for any causal and finite-energy signal or system allows for linear-in-parameter models for many types of system identification and approximation schemes, including adaptive filtering, both for fixed and non-fixed pole structures. Here we address only the "prototype" least-square (LS) approach to approximation, implied by the orthonormal Fourier series expansion with respect to functions (1).

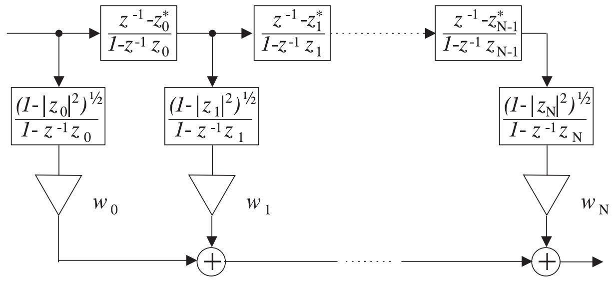

Figure 1: The Kautz filter. For $z_i = 0$ in (1) it degenerates to an FIR filter and for $z_i = a, -1 < a < 1$, it is a Laguerre filter where the tap filters can be replaced by a common prefilter.

A Kautz filter produces real tap output signals only in the case of real poles. However, from a sequence of real or complex conjugate poles it is always possible to form real orthonormal structures. From the variety of possible solutions it is sufficient to use the intuitively simple structure of Fig. 2, proposed by Broome: the second-order section outputs of Fig. 2 are *orthogonal* from which an orthogonal tap output pair if formed [2]. Normalization terms are completely determined by the corresponding pole pair $\{z_i, z_i^*\}$ and are given by $p_i = \sqrt{(1-\rho_i)(1+\rho_i-\gamma_i)/2}$ and $q_i = \sqrt{(1-\rho_i)(1+\rho_i+\gamma_i)/2}$, where $\gamma_i = -2RE\{z_i\}$ and $\rho_i = |z_i|^2$ can be recognized as corresponding second-order polynomial coefficients. The construction works also for

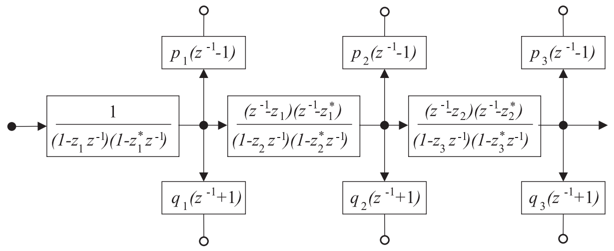

Figure 2: One realization for producing real Kautz functions from a sequence of complex conjugate pole pairs.

real poles but we use an obvious mixture of first- and second-order sections, if needed.

2.1. Kautz filter design

Kautz filter design can be seen as a two-step procedure involving the choosing of a particular Kautz filter (i.e., the poles) and the evaluation of the corresponding filter weights. For the latter, and in the case of a given target response $h(n)$ or $H(z)$, we use simply the Fourier coefficients, $c_i = (h, g_i) = (H, G_i)$, easily obtained by feeding the signal $h(-n)$ to the Kautz filter and reading the tap outputs $x_i(n) = G_i[h(-n)]$ at $n = 0$: $c_i = x_i(0)$. This implements convolutions by filtering and it can be seen as a generalization of rectangular window FIR design.

The contrast between the easy and well-defined model parameterization task and the complicated and non-linear model selection problem makes it tempting to use sophisticated guesses and random or iterative search in the pole position optimization. As a more analytic approach, the whole idea in the Kautz concept is how to incorporate desired *a priori* information to the Kautz filter. This may mean knowledge on system poles or resonant frequencies and corresponding time-constants, or indirect means, such as all-pole or pole-zero modeling. Furthermore, we have adopted a method proposed originally to pure FIR-to-IIR filter conversion [1], to the context of Kautz filter pole optimization. It resembles the iterative Steiglitz-McBride method of pole-zero modeling, but it genuinely and effectively optimizes (in the LS sense) the pole positions of a real Kautz filter, producing unconditionally stable and (theoretically globally) optimal pole sets for a desired filter order. In this paper we use the above *BU-method* as such or combined with, e.g., warped design or manual tuning of poles.

3. AUDIO APPLICATION EXAMPLES

We demonstrate the applicability of Kautz filter design in two different types of audio-oriented applications. The first one is the loudspeaker equalization task where frequency resolution is distributed both globally and locally. In the second case we use Kautz filters to model the body response of an acoustic guitar where the lowest frequencies are of primary interest.

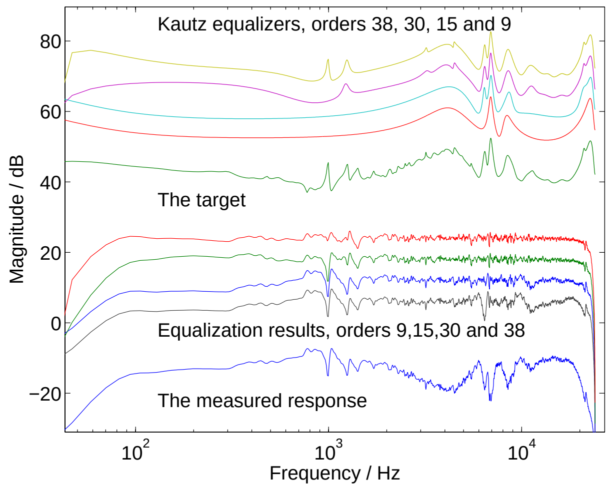

Figure 3: Kautz equalizers and equalization results for orders 9, 15, 30 and 38, compared to the measured loudspeaker response and the equalizer target response.

3.1. Example 1: Loudspeaker equalization

An ideal loudspeaker has a flat magnitude response and a constant group delay. Simultaneous magnitude and phase equalization would be achieved by modeling the response and inverting the model, or by identifying the overall system of the response and the Kautz equalizer, but here we demonstrate the use of Kautz filters in pure magnitude equalization, based on an inverted target response. The measured loudspeaker magnitude response and a derived equalizer target response are included in Fig. 3. The sample rate is 48 kHz.

As is well known, FIR modeling has an inherent emphasis on high frequencies on the auditorily motivated logarithmic frequency scale. Warped FIR (or Laguerre) [3] filters release some of the resolution to the lower frequencies, providing a competitive performance with 5 to 10 times lower filter orders than with FIR filters [4]. However, the filter order required to flatten the peaks at 1 kHz in our example is still high, of the order 200, and in practice Laguerre models up to order 50 are able to model only slow trends in the response. The proposed BU-method provides good pole sets for orders at least up to 40 and in Fig. 3 we have presented Kautz equalizers and equalization results for orders 9, 15, 30 and 38. For orders above 15, the BU-method produces poles really close to z=1 and omitting some of these poles actually tranquilize the low frequency region.

To improve the modeling at 1 kHz, we added three to four

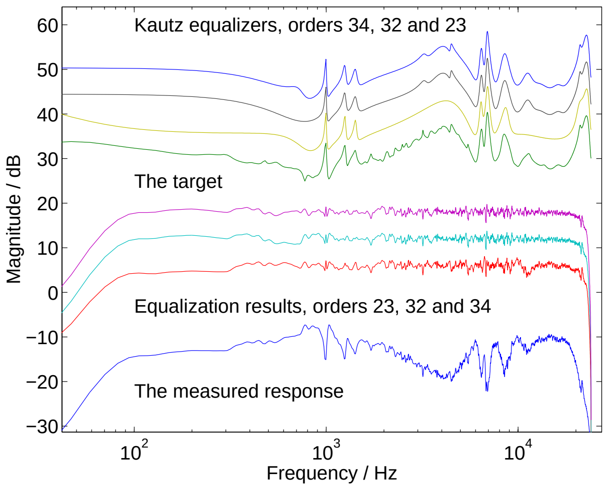

Figure 4: Kautz equalizers and equalization results for orders 23, 32 and 34, with combinations of manually tuned and BU-poles.

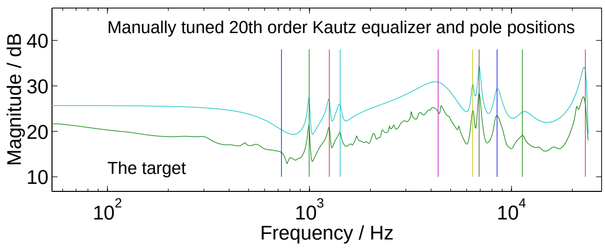

Figure 5: Manually tuned 20th order Kautz equalizer and the target magnitude response.

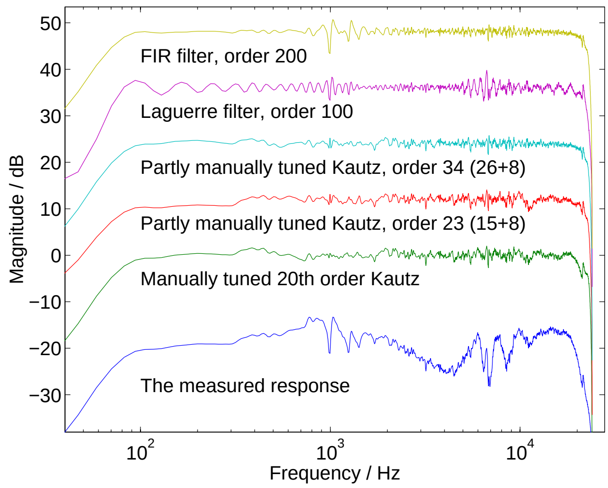

Figure 6: Comparison of FIR, Laguerre, and Kautz equalization results.

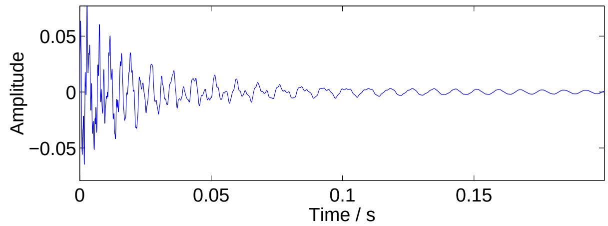

Figure 7: The measured impulse response of an acoustic guitar body.

manually tuned pole pairs to the BU-pole sets, corresponding to the resonances in the problematic area. Results for final filter orders 23, 32 and 34 are displayed in Fig. 4.

Finally, we abandon the pole sets proposed by the BU--method and try to tune 10 pole pairs manually to the target response resonances. The design is based on 10 selected resonances, represented with 10 distinct pole pairs, chosen and tuned to fit the magnitude response (Fig. 5).

A comparison of equalization results for some of the previous Kautz equalizers, and those achieved with FIR and Laguerre equalizers of orders 200 and 100, respectively, is presented in Fig. 6.

3.2. Example 2: Acoustic guitar body modeling

As another example of Kautz modeling we approximate a measured acoustic guitar body response sampled at 24 kHz (Fig. 7). The obvious disadvantage of a straightforward FIR filter implementation is that modeling of the slowly decaying lowest resonances requires a very high filter order. All-pole or pole-zero modeling are the traditional choices in improving the flexibility of the spectral representation. However, model orders remain problematically high and the basic design methods seem to work poorly. Perceptually motivated warped counterparts of all-pole and pole-zero modeling pay off, even in technical terms [5], but here we want to focus the modeling resolution more freely.

Figure 8 demonstrates that the BU-method is able to capture essentially the whole resonance structure. The Kautz filter order is 102 and the poles are obtained from a 120th order BU-pole set, omitting some poles close to $z = -1$. Lower-order models are achieved, e.g., by further pruning of the pole set.

Especially in this case of a target response dominated by the low-frequency part, we may compose very low order Kautz models with a combination of warping and BU-method: the BU-method is first applied to the *warped target response* [3] and then the poles are mapped back to the original frequency domain according to the *inverse allpass transformation*. In Fig. 9 are presented the magnitude responses of the attained Kautz models for orders 10, 16, 20 and 40,

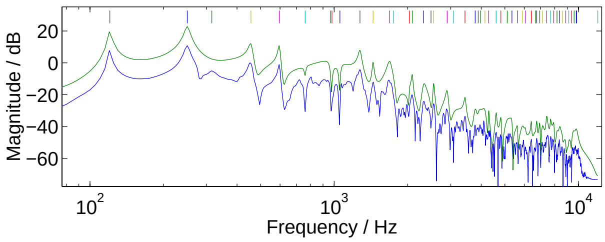

Figure 8: A 102th order Kautz model and the target magnitude response, and vertical lines indicating BU pole pair positions.

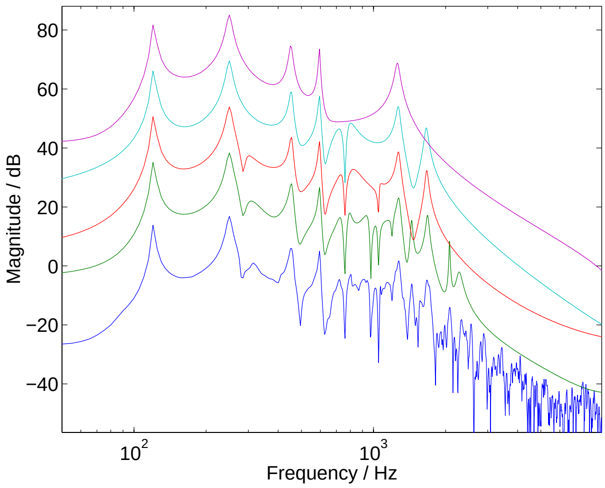

Figure 9: Displayed with offset from top to bottom, Kautz models of orders 10, 16, 20 and 40, and the target magnitude response.

where we used (allpass) warping parameter $\lambda = 0.7$. It is quite surprising that the BU-method found the five prominent resonances at model order 10, i.e., with exactly five complex conjugate pole pairs, in contrast to the unwarped case, where the required filter order is about 100.

Finally, in Fig. 10 we demonstrate that good fit to the five prominent resonances of the 10th order Kautz filter of Fig. 9 means also good match in the time-domain.

4. CONCLUDING REMARKS

We have demonstrated the potential applicability of Kautz filters in some typical audio signal processing tasks. They are found flexible generalizations of FIR and Laguerre filters, providing IIR-like spectral modeling capabilities with well-known favorable properties resulting from the orthonormality. A more detailed presentation of the underlying theory and the merely stated audio application results can be found in other related `~/publications` at

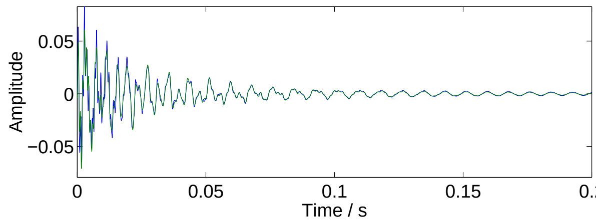

Figure 10: The impulse response of the 10th order Kautz filter compared to the measured response.

`http://www.acoustics.hut.fi` as well as MATLAB scripts and demos in `~/software/kautz`.

5. ACKNOWLEDGEMENTS

This work has been supported by the Academy of Finland as a part of project “Sound source modeling”.

6. REFERENCES

[1] H. Brandenstein and R. Unbehauen, “Least-Squares Approximation of FIR by IIR Digital Filters”, *IEEE Trans. Signal Processing*, vol. 46, no. 1, pp. 21–30, 1998.

[2] P. W. Broome, “Discrete Orthonormal Sequences”, *Journal of the Association for Computing Machinery*, vol. 12, no. 2, pp. 151–168, 1965.

[3] A. Härmä, M. Karjalainen, L. Savioja, V. Välimäki, U. K. Laine and J. Huopaniemi, “Frequency warped signal processing for audio applications”, *J. Audio Eng. Soc.*, vol. 48, no. 11, pp. 1011–1031, 2000.

[4] M. Karjalainen, E. Piirilä, A. Järvinen and J. Huopaniemi, “Comparison of loudspeaker equalization methods based on DSP techniques”, *J. Audio Eng. Soc.*, vol. 47, no. 1/2, pp. 15–31, 1999.

[5] M. Karjalainen and J. O. Smith, “Body modeling techniques for string instrument synthesis”, *Proc. Int. Computer Music Conf.*, Hong Kong, Aug. 1996, pp. 232–239.

[6] W. H. Kautz, “Transient Synthesis in the Time Domain”, *IRE Trans. Circuit Theory*, vol. CT–1, pp. 29–39, 1954.

[7] Y. W. Lee, *Statistical Theory of Communication*. John Wiley and Sons, New York, 1960.

[8] J. O. Smith and J.S. Abel, “Bark and ERB bilinear transform”, *IEEE Trans. Speech and Audio Processing*, vol. 7, no. 6, pp. 697–708, 1999.

[9] J. L. Walsh, *Interpolation and Approximation by Rational Functions in the Complex Domain*, 2nd Edition. American Mathematical Society, Providence, Rhode Island, 1969.