EFFICIENT SYMBOL SYNCHRONIZATION TECHNIQUES USING VARIABLE FIR OR IIR INTERPOLATION FILTERS

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ABSTRACT

Maximum Likelihood estimation theory can be used to develop optimal timing recovery schemes for digital communication systems. Tunable digital interpolation filters are commonly used for symbol timing adjustment. The so-called gathering structure offers an efficient and flexible realization structure for such an interpolation task. In this paper we propose a feedforward timing estimation scheme efficiently utilizing the tunable gathering filter structure for FIR filters and leveraging an algorithm previously developed for the so-called Farrow structure. The scheme is based on a low-order polynomial approximation of the log likelihood function. Furthermore, the scheme is extended for the IIR gathering structure with transient suppression, and the performance of FIR and IIR implementations and their computational complexity are compared in example simulations.

1. INTRODUCTION

The new trend is to use digital receivers where the sampling of the demodulated (baseband) signal is performed by a fixed sampling rate oscillator. This new design approach reduces the required analog components as most of the receiver functions are then performed digitally. This in turn increases flexibility, configurability, and integrability of the receiver contributing to the software radio concept which is the natural progression of digital radio receivers towards multimode, multistandard terminals in which the radio functionalities are defined by software.

This paper addresses the task of digital symbol synchronization in a digital receiver. Examples of previously published articles on developing symbol timing estimation algorithms using FIR filters include [2, 3, 5, 9]. IIR filters, however, often provide more efficient implementation than their FIR counterparts though they may suffer from transients caused by sudden coefficient changes. In this paper we propose novel efficient implementations for Maximum Likelihood (ML) [5] symbol timing estimation and synchronization utilizing the so-called gathering structures [6] for both FIR and IIR filters.

The object of this paper is to show if the IIR implementation for synchronization can match up to the FIR implementation in terms of performance and computational complexity. In practice, simple implementations are usually the best, and thus we will evaluate the performance of low-order FIR and IIR FD filters, both having the maximally flat group delay property, namely the Lagrange and the Thiran FD approximations [10].

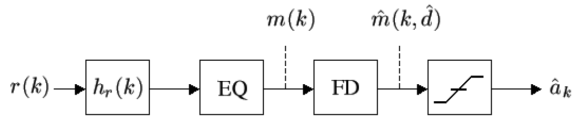

Figure 1: *Digital receiver using interpolation for symbol synchronization.*

The paper is organized as follows: In Section 2 the fixed sampling rate receiver model is viewed. In Section 3 the ML timing estimation and synchronization scheme presented in [5] is given a novel more efficient implementation using both FIR and IIR gathering structures. The computational complexities and synchronization performance of the different FIR and IIR implementations are evaluated in Section 4 and finally, Section 5 concludes the paper.

2. A SCHEME FOR ML TIMING ESTIMATION

Fig. 1 shows a receiver structure where symbol synchronization is performed digitally using a fractional-delay (FD) filter. The received signal $r(k)$ has been first digitally sampled at a fixed sampling rate $F_S = 1/T_S$. After sampling it is then passed through a matched receive filter $h_r(k)$ and a channel equalizer (EQ). Before symbol detection the timing offset is corrected using a FD interpolation filter and ML feed-forward timing estimation. Having the matched filter outside the timing adjustment loop often allows using lower sampling rates than otherwise [4].

According to [1, 5] the ML estimate of the log likelihood function (LLF) for symbol timing estimation is given as

$$\Lambda(d) = \sum_{k=1}^{M} \hat{a}_k^* \hat{m}(k,d), \tag{1}$$

where $\hat{a}_k$ are the correct or estimated symbol values, $\hat{m}(k,d)$ the fractionally delayed output samples of the matched filter, d a fractional delay, and M the number of used past symbols. The delay is assumed to remain constant within the block of M symbols. In our discussion we will assume a training signal and thus the symbols in (1) are considered known, i.e., $\{\hat{a}_k\} \rightarrow \{a_k\}$.

The ML feed-forward fractional delay estimate $\hat{d}$ at sampling

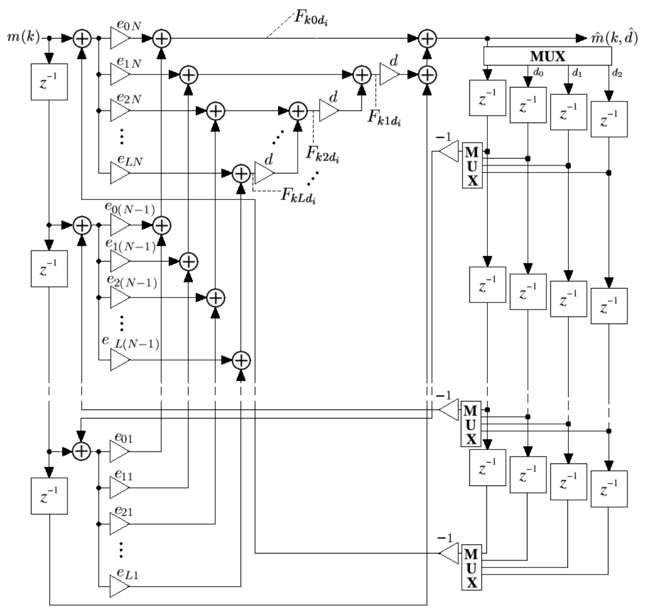

Figure 2: *Gathering structure with multiplexed output history for IIR allpass FD filters. The equations F_{kld_i} used in ML timing estimation are pointed out.*

rate is specified as

$$\hat{d} = \frac{\hat{\tau}}{T_S} = \arg\left\{\max_{\hat{d}}\{\Lambda(\hat{d})\}\right\} \tag{2}$$

where $\hat{\tau}$ is the timing error estimate in seconds. Provided we have an interpolation filter that can produce $\hat{m}(k, d)$ then $\Lambda(d)$ can then be evaluated for any chosen value(s) of d. For example, for a second-order polynomial approximation $\Lambda(d) \approx Ad^2 + Bd + C$ the maximum-point estimate $\hat{d}$ can be easily found in closed form using a fixed grid $\{d_i\}$ for $i \in \{0, 1, 2\}$ [3].

3. TIMING ADJUSTMENT USING GATHERING STRUCTURES

In this section we propose that the gathering structures introduced in [6] can be used efficiently for implementing tunable FD filters and simultaneously obtaining the values of the LLF in (1) at the desired values of d. The structures presented here are readily extendible for other orders P of polynomial approximation of $\Lambda(d)$.

3.1. FIR Structure

According to [6] we can express (1) as

$$\Lambda(d_i) = \sum_{k=1}^{M} \hat{a}_k^* M_{kd_i}, \qquad i = 0, 1, 2, \ldots, P \tag{3}$$

where

$$M_{kd_i} = \sum_{l=0}^{L} F_{kl} d_i^l = \sum_{l=0}^{L}\left[\sum_{n=0}^{N} m(k-n) c_{ln}\right] d_i^l \tag{4}$$

whose values F_{kl} may be obtained easily from an FIR gathering structure, in manner similar to what is shown for IIR gathering structures in Fig. 2.

3.1.1. Oversampling

If we oversample the input signal by an integer ratio O we end up with *O sets of candidate symbols* (polyphase components) from which *any one set* can be most optimum at a time. This means that we must evaluate

$$\Lambda_\lambda(d_i) = \sum_{k=1}^{M} \hat{a}_k^* M_{(Ok-\lambda)d_i}, \quad \text{for } \lambda = 0, 1, 2, \ldots, O-1 \tag{5}$$

for all the different polyphase components and choose the polyphase components for which $\Lambda_\lambda(\hat{d})$ is largest.

3.2. IIR Allpass Structure

This subsection proposes a novel approximation of the LLF (1) utilizing the IIR allpass gathering structure in [6] as

$$\begin{aligned}\Lambda(d_i) &= \sum_{k=1}^{M} \hat{a}_k^* \left[m(k-N) + \sum_{l=0}^{L} F_{kld_i} d_i^l\right] \\ &= \sum_{k=1}^{M} \hat{a}_k^* [m(k-N) + M_{kd_i}]\end{aligned} \tag{6}$$

where

$$F_{kld_i} = \sum_{n=1}^{N} [m(k-N+n) - \hat{m}_{d_i}(k-n)] e_{ln} \tag{7}$$

The peak point of the LLF is independent of $m(k-N)$, but the full form of Eq. (6) is preferred, as in this way the previously processed past samples can now be reused from memory. For signals oversampled by integer ratios the method introduced above in context with FIR filters in (5) applies for IIR filters as well.

The values F_{kld_i} may be obtained easily from the IIR gathering structure as shown in Fig. 2. The subscript d_i expresses the effect of the feedback in a recursive filter — the different delay values must be processed in separate feedback loops. Transient suppression [8] may be used to enhance the quality of the symbol estimates at the estimated delays $\hat{d}$.

3.2.1. Thiran g Allpass FD Filter

For “Thiran g” allpass FD approximation the equation (6) can be modified according to [7] in order to improve the quality of the FD approximation by incorporating the multiplier $g(d)$ into the loop as $M_{kd_i} \equiv g(d_i) \sum_{l=1}^{L} F_{kld_i} d_i^l$.

4. SIMULATIONS

The block size used for timing estimation was $M = 64$ symbols. The total number of transmitted symbols was $K = 256$ QAM-64 (RRC, $\alpha = 0.35$) symbols and the results were obtained by evaluating the $E = 128$ symbols received after the intial transient over a $SNR = 20$ dB channel. In order to keep things as simple as possible, an oversampling ratio of $O = 2$ was used. The LLF (3) was approximated using a $P = 3^{\text{rd}}$-order polynomial as in [5].

In order to compare the performance of FIR and IIR FD filters we implemented a Lagrange [10] FIR FD filter using the gathering structure, a Thiran g IIR FD filter as explained in [7], and also a "Thiran (polyn.)" whose constant coefficients are a polynomial approximation (in d) of the ideal Thiran allpass filter coefficients [10].

Table 1 shows the implementation complexities of the different FD filters in terms of delay elements, variable multipliers, and constant coefficients. For handling the block of M previously received symbols, a variety of implementations is available, but in order to save on computation we chose the reusability approach in which the previously computed symbol value estimates are stored into a bank of $(M-1)(P+1)$ delay elements. Using this approach the excess implementation complexity from block-processing of the M symbols is the same for both FIR and IIR, and thus not shown in Table 1.

Table 1: *Implementation complexity: Number of delay elements, number of variable multipliers, and number of constant coefficients, versus measured performance (the excess MSE value in brackets is measured using transient suppression).*

FD Filter Type	z^{-1}	d	$\{c_{lk}\}$ $\{e_{lk}\}$	max MSE	avg MSE
Thiran g				[dB]	[dB]
$L = N$					
$N = 1, I = 5$	2	6	4	-21(-21)	-34(-34)
$N = 2, I = 5$	4	7	10	-26(-29)	-39(-40)
$N = 3, I = 5$	6	8	13	-21(-39)	-42(-42)
Thiran (polyn.)					
$N = 1, L = 2$	2	2	3	-21(-21)	-35(-34)
$N = L = 2$	4	2	6	-20(-30)	-38(-39)
$N = L = 3$	6	3	12	-19(-39)	-42(-42)
Lagrange					
$N = L = 2$	2	2	9	-24	-30
$N = L = 3$	3	3	16	-23	-46

4.1. Results

In our simulations we sweeped the delay value (relative to the symbol rate) from $d = -0.5$ to $d = 0.5$ in 40 steps. For each delay estimate we measured the timing jitter variance Var$\{\hat{d} - d\}$, the timing jitter mean Avg$\{\hat{d} - d\}$, and the excess MSE caused by the interpolation relative to ideal synchronization as

$$MSE = 10\log_{10}(\text{Avg}\{|\Delta|^2\}/P_m), \tag{8}$$

where $\Delta = \hat{m}(k, \hat{d}) - m(k, d)$ and $P_m = \text{Avg}\{|m(k, d)|^2\}$.

Peak and average MSE measurements show that the second-order Lagrange FD filter has performance comparable to a first-order Thiran FD filter (see Table 1). Performance measurement results for different orders of the filters are plotted in figures 3, 4, and 5. In general the excess MSE of the allpass filter varies less over different values of d than the MSE of the Lagrange FIR FD filter.

In order to simplify the IIR implementation we compared the Thiran g performance against a polynomially approximated Thiran. As it turns out, the performance of this Thiran (polyn.) is

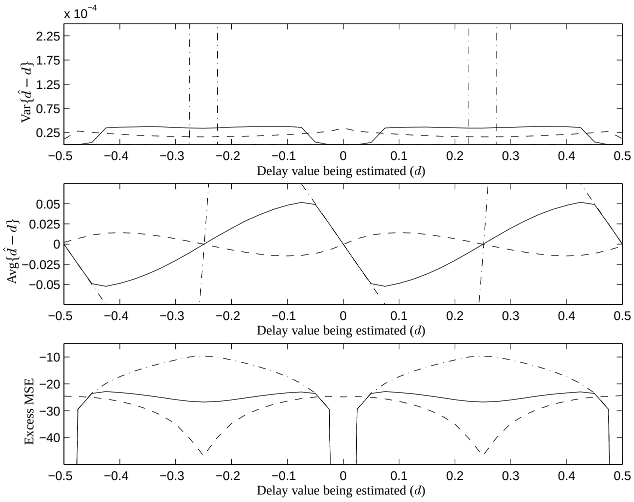

Figure 3: *Timing jitter variance, timing jitter mean, and excess MSE of the Lagrange FIR gathering FD filter. The dash-dot lines represent a first-order ($N = 1$), the dashed lines a second-order ($N = 2$), and the solid line a third-order ($N = 3$) implementation, respectively.*

similar to the Thiran g (see Fig. 5) while the implementation complexity of a first-order polynomially approximated Thiran without transient suppression is even less than that of an equivalent (second-order) Lagrange FIR FD filter (see Table 1). Using the IIR filters without transient suppression results in only minor degradation in synchronization performance as can be seen in Fig. 6.

5. CONCLUSIONS

In this paper we derived both FIR and IIR gathering structures for ML symbol synchronization. The synchronization performance was measured for both FIR and IIR FD implementations with different computational complexities. Furthermore, the IIR FD filters' performance is shown both with and without transient suppression indicating that when used for a synchronization application, it is not necessary to complicate the IIR design with transient suppression circuitry.

We conclude that IIR FD filters are a worthwile choice for synchronization applications. For higher-order implementations, models other than the Thiran FD approximation could be used because its performance improves only very little with increasing filter orders.

It remains a future research topic to extend the used ML estimation scheme for noninteger oversampling ratios. We note, however, that when d varies a lot the transient energy in IIR filters grows. This might eventually render the transient-suppressed IIR FD filters overcostly for such applications.

6. REFERENCES

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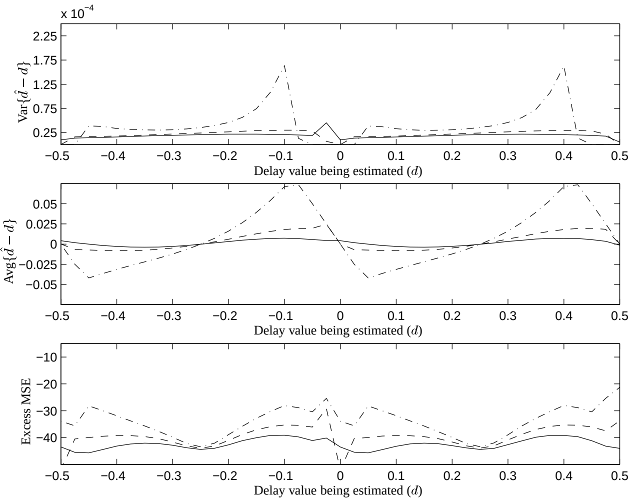

Figure 4: *Timing jitter variance, timing jitter mean, and excess MSE of the Thiran g IIR gathering FD filter. The dash-dot lines represent a first-order ($N = 1$), the dashed lines a second-order ($N = 2$), and the solid line a third-order ($N = 3$) implementation, respectively.*

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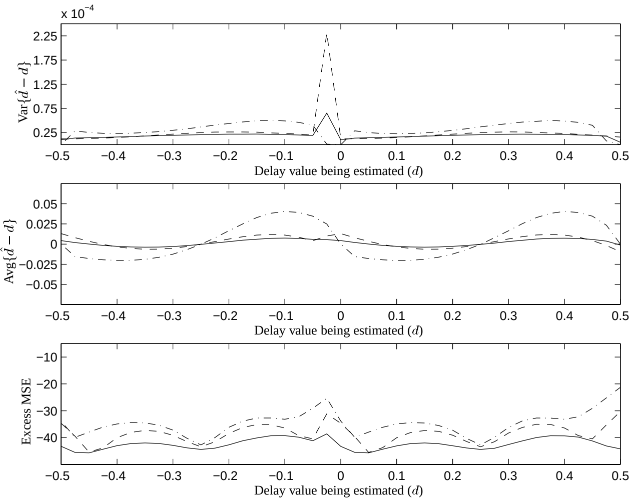

Figure 5: *Timing jitter variance, timing jitter mean, and excess MSE of the Thiran (polyn.) IIR gathering FD filter. The dash-dot lines represent a first-order ($N = 1$), the dashed lines a second-order ($N = 2$), and the solid line a third-order ($N = 3$) implementation, respectively.*

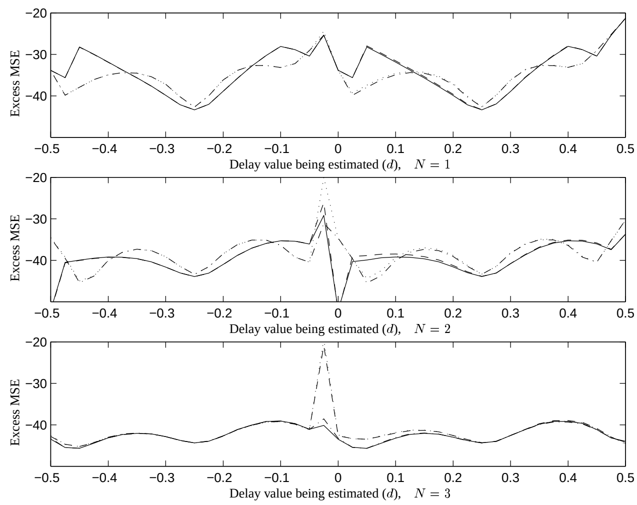

Figure 6: *Excess MSE of different orders of the IIR FD filters with and without transient suppression; IIR filter order $N = 1$ on top, below it $N = 2$ and $N = 3$. Solid and dashed lines represent the Thiran g with and without transient suppression, respectively. Dash-dotted and dotted lines represent the Thiran (polyn.) with and without transient suppression, respectively.*